Eur. Phys. J. B **13**, 203–204 (2000)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

Rapid Note

A stability criterion for financial markets

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Received 25 September 1999 and Received in final form 2 October 1999

Abstract. Using the theory of random cluster models, we give a stability criterion for financial markets with random communications between agents.

PACS. 02.50.-r Probability theory, stochastic processes, and statistics – 05.70.-a Thermodynamics – 89.90.+n Other topics of general interest to physicists

During the last two years an increasing number of physical concepts and models have been applied in the study of financial markets [3,7,10,12,13]. In the present note, we show how one can use some recent rigorous results on random cluster models to obtain a market stability criterion.

The random cluster models have been discovered in the seventies by Fortuin and Kasteleyn [8] and provide a general framework containing as special cases the standard statistical physics processes (percolation, Ising and Potts models). After the definition of the model we show how the rigorous results of [9,2] can be applied to give a criterion of the stability of financial markets with random communications between agents.

In general, a random graph model $G_n(p)$ consists of all graphs of n vertices, with vertex set $V = \{1, 2, \cdots, n\}$ and in which the edges are chosen independently and with probability p $(0 < p < 1)$. The edge-sets E are subsets of the set of $C_n^2 \equiv \binom{n}{2}$ pairs of elements of V.

The measure

$$\mu_{n,p}(E) = \prod_{e \in E} p^{|E|}(1-p)^{C_n^2 - |E|},$$

defined on the set E, describes the distribution of the random graph $G_n(p)$.

A number of classical results on this measure are due to Erdős and Rényi. The interested reader can find the general theory of random graphs and recent refinements in [1].

Starting from the previous random graph measure, we define the random cluster measure by

$$\mu_{n,p,q}(E) = \prod_{e \in E} p^{|E|}(1-p)^{C_n^2 - |E|} q^{\kappa_n(p,q)},$$

where $q > 0$ and $\kappa_n(p, q)$ denotes the number of components of the graph. For $q = 1$, the random cluster measure reduces to the Erdős-Rényi random graph model. If q is a positive integer the random cluster model corresponds to a q-state Potts model with interaction $J = -\log(1 - p)$. Moreover, in this case we can assume that the probability p becomes a function of q.

Before presenting the criterion let us relate the random cluster model to a financial market. The vertex set V represents the agents. The trading activity of each agent can be modelled by the parameter q; the simplest case we shall consider here is when $q = 3$. This corresponds to the three possibilities an agent can have either to buy, sell or not to trade. A reasonable assumption is to consider that the q values are uniformly distributed. The random graph structure can model the possible communications between agents. Each agent has a probability $p(q)$, depending on q, to align his action with any other agent. Thus, coalitions between agents could nicely be interpreted by the components of the random graph. Inside each coalition all agents have the same behaviour (*i.e.* the vertices of each component have the same value of q). Moreover, the total number of agreements between agents is given by the number of edges of $G_n(p, q)$.

Having this in mind we can *define the market as stable if the total number of agreements between agents remains smaller than a critical value. This value corresponds to the appearance of a giant component in the random graph* $G_n(p, q)$ (the giant coalition leads to a market boom or krach).

The case $q = 1$ has been used in [6] for the study of the herd behaviour and aggregate fluctuations and recently generalised in [14,4,15,5,16]. Although in the introduced model the parameter q equals to 3, all estimations are valid only for $q = 1$. Indeed, the probability p considered in [6] has the form $\frac{c}{n}$ where the constant c has values close to

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and smaller than 1 (the critical value of c being 1). This is the case for $q = 1$; for different values of q the parameter c is assumed to be a function of q. In our case, $q = 3$ and $p = \frac{c(q)}{n}$. The critical value of $c_{cr}(q)$ is defined [2] by

$$c_{cr}(q) = 2\frac{q-1}{q-2}\log(q-1) = 4\log 2 = 2.77.$$

It can be shown, using the techniques of[1] and [2], that if $c(q) < c_{cr}(q)$ (subcritical case), the random graph $G_n(p, q)$ comprises only trees and unicyclic components. Moreover, in the limit $n \to \infty$, the number of edges in $G_n(p, q)$ is $\frac{c(q)n}{2q} + o(n)$. We omit here the rather technical proof.

We can now give the stability criterion: Consider a stock market with n agents trading in a given asset. Assume that each agent has the possibility either to buy, sell the stock or not to trade. Moreover, each agent can agree with other agents to have the same trading behaviour. *We define the market as stable if the total number of agreements between the n agents is always smaller than* $0.46n$. (This corresponds indeed to the number of edges of the graph for $c(q) = c_{cr}(q)$).

It should be noticed that the estimations based on the random graph theory are valid in the limit $n \to \infty$. Trading agents being finite sets, there is a certain amount of ambiguity about the way in which the proposed models can be applied. However, in the main active markets the number of traders is sufficiently big to guarantee that such results can be used.

Let us note that the theory of the random cluster model on a complete graph is valid not only for integer values of the parameter q. Indeed, this could be used for the study of more realistic situations where the agents are allowed to fix a set of prices. In a forthcoming paper we will consider this case and we will use large deviation theory to calculate also the number of agent coalitions (components of the graph) [11].

This work was partially supported by the HCM grant CHRX-CT93-0411.

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